IB Mathematics SL Revision Checklist

V1.0 August 2017

1 Algebra

☐ Find common difference(d), first term(u_1), nth term(u_n), sum of n terms(S_n) in an arithmetic sequence.

☐ Find common ratio(r), first term(u_1), nth term(u_n), sum of n terms(S_n), sum to infinity(S_∞) in a geometric sequence. Remember that a geometric series converges only when $-1 < r < 1$.

☐ Recognise arithmetic and geometric patterns in real world problems including population growth and compound interest.

☐ Apply properties of sigma notation to solve series problems.

$$\sum_{k=1}^{n} a = an \qquad \sum_{k=1}^{n}(ak) = a\sum_{k=1}^{n} k \qquad \sum_{k=1}^{n}(ak+b) = a\sum_{k=1}^{n} k + \sum_{k=1}^{n} b \quad \text{where } a \text{ and } b \text{ are constants.}$$

☐ Use algebraic simplification methods such as expansion and factorisation to solve problems.

$$a(b+c) = ab+ac \quad (a+b)(c+d) = ac+ad+bc+bd \quad (a\pm b)^2 = a^2 \pm 2ab + b^2 \quad a^2-b^2 = (a+b)(a-b)$$

☐ Solve simultaneous equations using substitution, elimination or using graphical method using GDC.

☐ Use rules of exponents to solve problems.

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad a^m = \frac{1}{a^{-m}} \qquad a^0 = 1 \qquad (a^m)^n = a^{mn} = a^{nm}$$

☐ Convert between exponent notation and log notation. $a^x = b \Leftrightarrow x = log_a b$.

☐ Use rules of logarithms to solve problems.

☐ Use the general term ($(r+1)$th term) of a binomial expansion ${}^nC_r(a)^{n-r}(b)^r$ to find a given term.

☐ Reverse binomial expansion to find the terms a or b or the power n in $(a+b)^n$.

2 Functions and Equations

☐ Remember the shape of the parent functions when $a > 0$ and $a < 0$.

$$y = ax \qquad y = ax^2 \qquad y = ax^3 \qquad y = ax^{-1} = a\tfrac{1}{x} \qquad y = ax^{-2} = a\tfrac{1}{x^2} \qquad y = a^x \qquad y = a\log(x)$$

☐ Find **domain** (all the values that x can take) of a function using the three scenarios.

$$f(x) = \tfrac{1}{x-a} \Rightarrow x - a \neq 0 \qquad f(x) = \sqrt{x-a} \Rightarrow x - a \geqslant 0 \qquad f(x) = \log(x-a) \Rightarrow x - a > 0$$

☐ Find the vertical asymptotes of a function using the three scenarios above.

☐ Find the **range** (all the values that y can take) of a function.

☐ Find the horizontal asymptotes of a function by considering what happens to the function when $x \to \infty$. Remember the two common cases,

$$f(x) = \tfrac{1}{x-a} + b \quad \text{when} \quad x \to \infty, f(x) \to b \qquad f(x) = a^{-x} \quad \text{when} \quad x \to \infty, f(x) \to 0$$

☐ Find inverse of a function, $f^{-1}(x)$. Swap x and y then make y the subject.

☐ Know that the graphs of the function and its inverse are mirror images on the line $y = x$. Use this information to solve problems.

☐ Find $(f \circ g)(x)$ given $f(x)$ and $g(x)$.

☐ Find points of intersection when two functions $f(x)$ and $g(x)$ intersect - Equate the two functions and solve manually or use GDC (paper 2).

☐ Know the eight types of transformations of graphs - y-translation ($\uparrow$ or $\downarrow$), x-translation ($\leftarrow$ or $\rightarrow$), y-stretch $\left(\begin{smallmatrix}\uparrow\\ \downarrow\end{smallmatrix}\right.$ or $\left.\begin{smallmatrix}\downarrow\\ \uparrow\end{smallmatrix}\right)$, x-stretch ($\leftarrow \rightarrow$ or $\rightarrow \leftarrow$).

☐ Solve quadratic equations using factorisation, completing square method, formula or by graphical method using GDC (paper 2)

☐ Find the x-intercept(zeros or roots), y-intercept of a quadratic graph.

☐ Find the line of symmetry and the coordinates of the vertex in all three forms of the quadratic equation.

	$ax^2 + bx + c$	$a(x-p)(x-q)$	$a(x-h)^2 + k$
Line of symmetry	$x = \frac{-b}{2a}$	$x = \frac{p+q}{2}$	$x = h$
Coordinates of vertex	$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	$\left(\frac{p+q}{2}, f\left(\frac{p+q}{2}\right)\right)$	(h, k)

3 Circular functions and trigonometry

☐ Find arc length, area of a sector and use it to solve problems.

☐ Find the three ratios sine, cosine and tan for any angle using 'basic angle' in each quadrant.

☐ Know the sign of each ratio in each quadrant - **A**ll **S**ilver **T**ea **C**ups.

☐ Remember the identities that apply in each quadrant.

1st $\rightarrow \sin(90^\circ - \theta) = \cos(\theta)$ and $\cos(90^\circ - \theta) = \sin(\theta)$ **2nd** $\rightarrow \sin(180^\circ - \theta) = \sin(\theta)$

3rd $\rightarrow \tan(180^\circ + \theta) = \tan(\theta)$ **4th** $\rightarrow \cos(-\theta) = \cos(\theta)$

☐ Know the trigonometric ratios for common angles $(0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ)$ by memory (paper 1).

☐ Use inverse trigonometric values to find angles within a given domain.

☐ Solve problems using trigonometric identities (Pythagorean identity, double angle formula).

☐ Know the shape of $f(x) = \sin x$, $f(x) = \cos x$ and $f(x) = \tan x$ graphs.

☐ Use transformations on general sine and cosine function $f(x) = a\sin(b(x-c)+d$. Know how to find the values of amplitude(a), period($\frac{2\pi}{b}$), x-translation(c) and y-translation(d) using given coordinates.

☐ Solve real world problems using the knowledge of general sine and cosine functions including Ferris wheels, water wheels, movement of tides and buoys.

☐ Approximate a circular function using 'sine regression' function in your GDC (paper 2).

☐ Solve problems using sine rule, cosine rule and area of triangle.

☐ Solve real world problems using bearings, angle of elevation and angle of depression.

4 Vectors

☐ Represent the path between two points using multiples of $\overrightarrow{a}$, $\overrightarrow{b}$.

☐ Find the vector $\overrightarrow{AB}$ given the coordinates of A and B.

☐ Switch between column vector form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and unit vector form $(i + j + k)$.

☐ Find the **magnitude** (size) of a vector.

☐ Find the **unit vector** (a vector that has a magnitude of 1) of a vector.

☐ Find the scalar product (dot product) of two vectors and use it to find angle between two vectors. Know that for perpendicular vectors the dot product is zero.

☐ Write the vector equation of a line using the direction of the line and a point through which it passes.

☐ Know the properties of basic shapes and use vector algebra for proofs.
Triangles - equilateral, isosceles, right angled and scalene.
Quadrilaterals - square, rectangle, trapezium, parallelogram, rhombus and kite.
Regular polygons - pentagon, hexagon, heptagon, octagon etc.

☐ Switch between vector form of a line and Cartesian form of a line.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = t \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \qquad \text{and} \qquad \frac{x - b_1}{a_1} = \frac{y - b_2}{a_2} = \frac{z - b_3}{a_3} = t$$

☐ Show if two lines are parallel, intersecting or skew.
Parallel lines - Direction vectors (of lines) are the same, one could be a scalar multiple of the other.
Intersecting lines - Values of parameter s and t will satisfy all three equations for x, y and z.
Skew lines - Directions are different and there are no values of the parameters s and t that will satisfy all three equations for x, y and z.

☐ Show that given three points A, B, C are co-linear. Find the directional vectors of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and show that they are equal.

☐ Know the relationship between displacement, velocity(direction),time and use it to solve problems.

$$\text{Final displacement} = \text{Initial displacement} + (\text{velocity vector}) \times \text{time}$$

☐ Know that the magnitude of the velocity vector gives speed.

5 Statistics and Probability

☐ Find the mean, median, mode of a set of data.

☐ Find mean($\bar{x}$), median, mode, standard deviation(σ), variance(σ^2) from frequency tables, grouped frequency tables using GDC (paper 2).

☐ Read and construct box & whisker plots, stem & leaf plots.

☐ Know what happens to the mean, standard deviation and variance when,

- ☐ The same value is added or subtracted to all data points.
  New mean = old mean ± value, standard deviation and variance remain the same
- ☐ When all data points are multiplied or divided by the same value.
  Multiplied - new mean = old mean×value, new s.d = old s.d×value, new var = old var×(value)^2
  Divided - new mean = old mean÷value, new s.d = old s.d÷value, new var = old var÷(value)^2

- ☐ Find first quartile(Q_1), second quartile or median(Q_2), third quartile(Q_3), percentiles and Inter Quartile Range or IQR($Q_3 - Q_1$) in cumulative frequency diagrams.
- ☐ Find Pearson's product-moment correlation coefficient(r) using GDC and describe the correlation of two data sets using appropriate keywords.
- ☐ Find the regression line $y = ax + b$ for two data sets and use it for estimations.
- ☐ Know when the estimations are reliable. Know the meaning of interpolation, extrapolation, outliers.
- ☐ Represent probability of multiple events using tree diagrams, grids and Venn Diagrams.
- ☐ Use Venn diagrams to represent intersection(and), union(or) and compliment(not) of events.
- ☐ Use probability formula to solve problems with mutually exclusive events, independent events and conditional probability.
- ☐ Calculate the expected value for discrete data and know that $E(X) = 0$ in a 'fair game'.
- ☐ Solve binomial probability problems manually or by using Binormpdf, Binormcdf in GDC (paper 2).
- ☐ Find the mean and variance of a binomial probability distribution.
- ☐ Know properties of normal distribution and solve problems using Normalcdf, Inversenorm in GDC.
- ☐ Find mean(μ) or standard deviation(σ) in a normal distribution using the z - distribution.

6 Calculus

- ☐ Know how to find limits of a function, limits to infinity.
- ☐ Know how to differentiate using first principles, using basic rules - general, chain, product and quotient.
- ☐ Use derivatives to find the nature of turning points.

	Maximum	Minimum	Point of inflexion
Sign change of slope	$+ \to -$	$- \to +$	$+ \to +$ or $- \to -$
$\frac{dy}{dx}$ or $f'(x)$ at x	0	0	any value
$\frac{d^2y}{dx^2}$ or $f''(x)$ at x	negative	positive	0

- ☐ Identify intervals for which a function is increasing ($\frac{dy}{dx} > 0$) or decreasing ($\frac{dy}{dx} < 0$).
- ☐ Use derivatives to find equations for velocity and acceleration given the equation for displacement. Use these equations to solve motion problems.
- ☐ Use turning points to optimise functions (find maximum or minimum values in a given problem).
- ☐ Integrate functions using basic rules - general, substitution or observation. Use boundary conditions given in the problem to find constant C.
- ☐ Find **definite integrals** (integrals with limits) manually or using GDC.
- ☐ Remember properties of integrals and use them to solve problems.
- ☐ Find area under a curve, area between curves using integrals. Know that area below the x axis is always negative.
- ☐ Find volumes of revolution when areas bounded by curves are rotated about the x axis or y axis.
- ☐ Use integration and boundary conditions to find equations for velocity and displacement given the equation for acceleration. Use these equations to solve motion problems.
- ☐ Use the area under the velocity-time graph to find displacement. Know that area above the x axis gives displacement in one direction and area below the x axis gives displacement in the opposite direction.