IB Mathematics SL
Paper 1

Mock Exam Revision Set 1

Learn Tuition Centre

ANSWER KEY

Section A

1. (a) $x > 0$ OR $\{x|x > 0\}$ OR $0 < x < \infty$ OR $]0, \infty[$ *A1*
Award 0 marks for $x \geqslant 0$.

(b) $g(2) = (2)^2 + 5 = 4 + 5 = 9$
Award 1 mark for correct substitution and answer. *A1*

(c) $g \circ f(x)$
$g(\frac{1}{\sqrt{x}}) = \frac{1}{x} + 5$ *M1A1*

(d) $\frac{1}{x} + 5 = 6x$
$1 + 5x = 6x^2$
$0 = 6x^2 - 5x - 1$ *M1*
$0 = (6x + 1)(x - 1)$ correct quadratic factorisation. *M1*
$x = 1$ or $x = -\frac{1}{6}$ *A1A1*
Award one mark for each correct answer.

2. (a) i. $1 - P(A \cap B) = p \Rightarrow 1 - 0.9 = p$
$p = 0.1$ *M1*

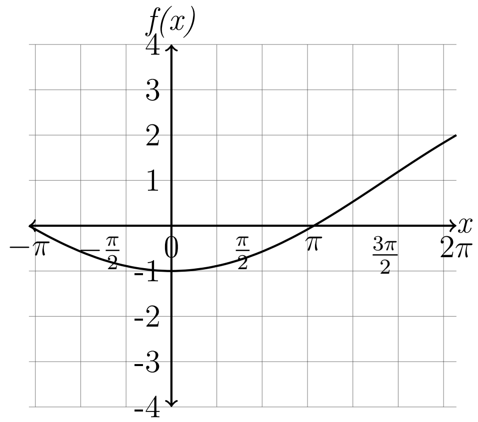
ii. $P(A) - P(A \cap B) = 0.3 - 0.2 = 0.1$ *M1*
$P(A \cup B) = 0.9 = 0.1 + 0.2 + q + 0.1$ *M1*
(OR any other method)
$q = 0.5$ *A1*

(b) $P(B) = 2p + q + 0.1 = 0.2 + 0.5 + 0.1$ *M1*
(OR any other method)
$P(B) = 0.8$ *A1*

3. (a) i. Amplitude $= -2$ *A1*
Period $= \frac{2\pi}{1/3} = 6\pi$ *A1*

ii. Recognising maximum is when $cos(\frac{\pi}{3}x + 2) = -1$ therefore $f(x) = -2(-1) + 1 = 3$ *A1*
Recognising minimum is when $cos(\frac{\pi}{3}x + 2) = 1$ therefore $f(x) = -2(1) + 1 = -1$ *A1*

(b)



Award one mark each for x intercepts and the highest value at $x = 2\pi$ *A1A1A1*

4. (a) Recognising axis of symmetry is at $x = 3.5$. *A1*

Use the form $f(x) = a(x - p)(x - q)$

$12 = a(0 - 3)(0 - 4)$

$a = 1$

$\therefore f(x) = x^2 - 7x + 12$ *A1*

Find k using,

$k = f(\frac{7}{2}) = (\frac{7}{2})^2 - 7(\frac{7}{2}) + 12$ *M1*

$\therefore f(x) = (x - \frac{7}{2})^2 - \frac{1}{4}$

(b) Minimum is at $(\frac{7}{2}, -\frac{1}{4})$ *A1*

(c) $-f(x) = -(x - \frac{7}{2})^2 + \frac{1}{4}$

$-(x - \frac{7}{2})^2 + \frac{1}{4} = -x$ *M1*

$-4(x - \frac{7}{2})^2 + 1 = -4x$

$-4x^2 + 32x - 48 = 0$

$4x^2 - 32x + 48 = 0$

$x^2 - 8x + 12 = 0$ *M1*

$(x - 2)(x - 6) = 0$

$x = 2, x = 6$ *A1*

5. METHOD 1

Recognise $BD = 1cm$

Attempt to find $\cos\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\cos\theta = \sqrt{1 - \sin^2\theta}$ *M1*

$\cos\theta = \sqrt{1 - \frac{1}{9}}$

$\cos\theta = \frac{\sqrt{8}}{3}$ *A1*

For triangle BDC,

$BC \times \cos\theta = BD$

$BC \times \frac{\sqrt{8}}{3} = 1$

$BC = \frac{3}{\sqrt{8}}$ *A1*

Use area formula for a triangle to calculate area

$A = \frac{1}{2} \times BD \times BC \times \sin\theta$

$A = \frac{1}{2} \times 1 \times \frac{3}{\sqrt{8}} \times \frac{1}{3}$ *M1*

$A = \frac{1}{2\sqrt{8}} = \frac{1}{4\sqrt{2}}$ *A1*

METHOD 2

Recognise $BD = 1cm$

Find AD using Pythagoras theorem

$AD = \sqrt{3^2 - 1^2} = \sqrt{8}$ *A1*

Attempt to find $\tan\theta$.

$\tan\theta = \frac{1}{\sqrt{8}}$

Use $\tan\theta$ or similarity of triangles to find DC.

$\tan\theta = \frac{BD}{AD} = \frac{DC}{BD}$

$\frac{1}{\sqrt{8}} = \frac{DC}{1}$ $M1$

$DC = \frac{1}{\sqrt{8}}$ $A1$

Use area formula for a triangle to calculate area

$A = \frac{1}{2} \times BD \times DC$

$A = \frac{1}{2} \times 1 \times \frac{1}{\sqrt{8}}$ $M1$

$A = \frac{1}{2\sqrt{8}} = \frac{1}{4\sqrt{2}}$ $A1$

6. (a) At the point of intersection the curves have the same coordinates.

$2e^{2k} + 3e^k + 2 = e^{2k} + 6$

$2(e^k)^2 + 2e^k - 4 = 0$ $M1$

Recognising quadratic form,

Let $e^k = m$

$2m^2 + 2m - 4 = 0$

$m^2 + m - 2 = 0$ $M1$

$(m-1)(m+2) = 0$

$m = 1$ or $m = -2$

$\therefore e^k = 1$ or $e^k = -2$(reject solution) $M1$

$k = 0$ $A1$

(b) At $k = 0, f(0) = 7$ or $g(0) = 7$ $M1$

$\therefore$ Coordinates $= (0, 7)$ $A1$

7. (a) $\log_3(xy) = \log_3 x + \log_3 y$

$\frac{\log_9 x}{\log_9 3} + \log_3 y$ (Recognising change of base) $M1$

$\frac{a}{\frac{1}{2}} + b$

$2a + b$ $A1$

(b) $\log_9 x = a \Rightarrow 9^a = x \Rightarrow 9^{2a} = x^2$ $M1$

$\log_3 y = b \Rightarrow 3^b = y$ $M1$

$x^2y = 9^{2a} \times 3^b = (3^2)^{2a} \times 3^b = 3^{4a} \times 3^b = 3^{4a+b}$ $A1$

Section B

1. (a) Recognising the arithmetic sequence $5, 8, 11, \ldots$ with common difference of 3 $M1$
$P_r = 5 + (r-1)3 = 3r + 2$ $A1$

(b) Recognising the arithmetic sequence $5, 6, 7, \ldots$ with common difference of 1 $M1$
$R_r = 5 + (r-1) = r + 4$ $A1$

(c) Make r the subject in equation in part (a) or in part(b) and substitute into the other equation $M1M1$
From part (b), $r = R_r - 4$
Substituting into equation in part(a), $P_r = 3(R_r - 4) + 2 = 3R_r - 10$ $A1$

(d) $P = 3 \times 16 - 10 = 48 - 10 = 38$ $A1$

(e) Use $S_n = \frac{n}{2}\{2u_1 + (r-1)d\}$
$S_{20} = \frac{20}{2}\{2(5) + (20-1)3\}$ $M1A1$
$S_{20} = 670$ $A1$

i. Radius of all flowers side by side, $S_{20} = \frac{20}{2}\{2(5) + (20-1)(1)\} = 240mm$ $M1$
Diameter of all flowers side by side, $240 \times 2 = 480mm$ $M1$
Length of the display board, $480 + 10 + 10 = 500mm$ $A1$
Allow only one mark if the radius is not multiplied by 2.

ii. Largest flower radius, $R_{20} = 20 + 4 = 24mm$
Largest flower diameter, $48mm$
Width of the display board, $48 + 20 = 68mm$ $A1$

2. (a) $f(x) = \frac{4\sin x\cos x}{(1+\sqrt{2}\sin x)(1-\sqrt{2}\sin x)}$
$= \frac{4\sin x\cos x}{1^2-(\sqrt{2}\sin x)^2}$
$= \frac{4\sin x\cos x}{1-2\sin^2 x}$
$= \frac{2\times 2\sin x\cos x}{1-2(1-\cos^2 x)}$
$= \frac{2\sin 2x}{1-2+2\cos^2 x}$
$= \frac{2\sin 2x}{2\cos^2 x-1}$
$= \frac{2\sin 2x}{\cos 2x} = 2\tan 2x$

(b) i. $f(x) = 2\tan(2x)$
$f'(x) = 2\sec^2(2x) \times 2$ $M1A1$
$= \frac{4}{\cos^2(2x)}$ $A1$

ii. $f'(\pi) = \frac{4}{\cos^2(2\pi)} = 4$ $A1$

(c) **METHOD 1**
Pick two points one to the left and one to the right of the turning point and check the sign of the slope at each point.
At $x = \frac{3\pi}{4}$,
$f'(\frac{3\pi}{4}) = \frac{4}{\cos^2(\frac{3\pi}{4})} = \frac{4}{\frac{1}{2}} = 8$ (positive).
At $x = \frac{3\pi}{4}$,

$f'(\frac{5\pi}{4}) = \frac{4}{\cos^2(\frac{5\pi}{4})} = \frac{4}{\frac{1}{2}} = 8$ (positive).
The sign of the slope does not change. $\therefore$ it is a point of inflection.

METHOD 2
Find the second derivative of $f(x)$ and show that at $x = 0$ it is equal to zero.
$f'(x) = 4(\cos(2x))^{-2}$
$f''(x) = 4(-2)(\cos(2x))^{-3} \times (-\sin 2x) \times 2$
$f''(x) = \frac{16\sin(2x)}{\cos^3(2x)}$
$f''(\pi) = \frac{16\sin(2\pi)}{\cos^3(2\pi)} = 0$
As the second derivative is equal to zero, $x = \pi$ is a point of inflection.

(d) $2\tan 2x = \sin 2x$
$2 \cdot \frac{\sin 2x}{\cos 2x} = \sin 2x$
$2\sin 2x = \sin 2x \cdot \cos 2x$
$2\sin 2x - \sin 2x \cdot \cos 2x = 0$
$\sin 2x(2 - \cos 2x) = 0$ *M1A1*
$\sin 2x = 0$ or $(2 - \cos 2x) = 0$ solution rejected
$2x = 0, \pi, 2\pi, 3\pi, 4\pi$
$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ *A1*

3. (a) i. Let $\frac{x-2}{4} = \frac{y+1}{2} = -z = t$ *M1*

$x = 2 + 4t$
$y = -1 + 2t$
$z = -t$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$ *A1*

ii. $A = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ *A1*

(b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -17 \end{pmatrix} + s\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ *A1*

(c) $\overrightarrow{AB} = \begin{pmatrix} 2p-2 \\ 3-(-1) \\ -2 \end{pmatrix} = \begin{pmatrix} 2p-2 \\ 4 \\ -2 \end{pmatrix}$
$|\overrightarrow{AB}| = \sqrt{(2p-2)^2 + 4^2 + (-2)^2} = \sqrt{84}$ *M1*
$\sqrt{4p^2 - 4p + 4 + 16 + 4} = \sqrt{84}$
$4p^2 - 4p - 60 = 0$
$p^2 - p - 15 = 0$
$(p+3)(p-5) = 0$
$p = 5$ *A1*
$\overrightarrow{BC} = \begin{pmatrix} 4-10 \\ q+3-3 \\ -18-(-2) \end{pmatrix} = \begin{pmatrix} -6 \\ q \\ -16 \end{pmatrix}$
$\overrightarrow{BC} = \sqrt{(-6)^2 + q^2 + (-16)^2} = \sqrt{308}$ *M1*

$36 + q^2 + 256 = 308$
$q^2 - 16 = 0$
$q = 4$ *A1*

(d) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \\ -2 \end{pmatrix} + r\begin{pmatrix} 4-10 \\ 7-3 \\ -18-(-2) \end{pmatrix}$ *M1A1*

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \\ -2 \end{pmatrix} + r\begin{pmatrix} -6 \\ 4 \\ -16 \end{pmatrix}$ *A1*

(e) $\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 4 \\ -16 \end{pmatrix}$ Attempt at scalar product *M1*

$= -24 + 8 + 16 = 0 \quad \therefore \quad A\hat{B}C = 90^\circ$ *A1*

(f) Area $= \frac{1}{2} \cdot \sqrt{84} \cdot \sqrt{308}$ *A1*
$= \frac{1}{2} \cdot \sqrt{4 \cdot 21} \cdot \sqrt{4 \cdot 77}$
$= 2\sqrt{21}\sqrt{77}$ *A1*